2017

Time : 3 hours
Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.
Answer any six questions, selecting at least one from each group, question no 1 is compulsory.

1. Fill up the blanks in the following : 2 × 10 = 20

   (a) $A \cap (B \cup C) =$ $(A \cap B) \cup (A \cap C)$

   (b) $A \times (B \cap C) =$ $(A \times B) \cap (A \times C)$

   (c) If A' and B' are the transposes of A and B respectively then $(AB)' =$ ____

   (d) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$, then rank of A= ___

   (e) Principal value of $Cosh^{-1}\omega$ = ____

   (f) The set of all feasible solution of a LPP is a ___

   (g) A set is called countable if it is finite or ___

   (h) The variables which are added to LHS of the constraints in a LPP to convert them into equalities are called the ___ variables.

   (i) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is divisible by $x - \alpha$

   (j) An equation $f(x) = 0$ cannot have more negative roots than the number of changes of sign in ___

Group – A

2. (a) Prove that
   $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$ 8

   (b) State and prove the fundamental theorem on equivalence relation. 8

3. (a) Define partially ordered set and totally ordered set. Prove that $(R, \leq)$ is a partially ordered set which is also totally ordered. 8

   (b) Prove that if A and B are countable sets, then A×B is also countable. 8

4. (a) If $\sin(\theta + i\phi) = u + iv$, prove that $\sin^2\theta$ and $\cosh^2\phi$ are the roots of the equation
   $t^2 - (1 + u^2 + v^2)t + u^2 = 0$ 8

   (b) If $\cosh(u + iv) = x + iy$, show that
   $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$ and $\frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$
   8

5. (a) Show that $\cos\theta = \prod_{r=1}^{\infty}\left[1 - \frac{4\theta^2}{(2r-1)^2\pi^2}\right]$ 8

   (b) Prove that $\frac{1}{1^3.2^3} + \frac{1}{2^3.3^3} + \frac{1}{3^3.4^3} + \ldots = 10 - \pi^2$ 8

Group – B

6. (a) If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then prove that

$$(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$$ 8

(b) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$. 8

7. (a) Find A^{-1}, if $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ 8

(b) If $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$, find rank of A. 8

Group – C

8. (a) Prove that the intersection of two convex set is also a convex set.

(b) Solve the LPP graphically

Max Z = $5x_1 + 7x_2$

Subject to

$x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 24$

$10x_1 + 7x_2 \leq 35, \quad x_1, x_2 \geq 0$ 8

9. Use simplex method to solve the following LPP problem

Max Z = $9x_1 + 7x_2$

Subject to

$x_1 + 2x_2 \leq 7$

$x_1 - x_2 \leq 4, \quad x_1, x_2 \geq 0$ 16

Group – D

10. (a) Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ the roots being in GP. 8

(b) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in AP and hence solve the equation $x^3 - 12x^2 + 39x - 28 = 0$. 8

11. (a) If α, β, γ be the roots of $x^3 - px^2 + qx - r = 0$, find the value of $\frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} + \frac{1}{\alpha^2\beta^2}$. 8

(b) Solve the cubic equation $x^3 - 9x + 28 = 0$. 8

12. (a) Find the condition that the roots $\alpha, \beta, \gamma, \delta$ of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ should be connected by the relation $\alpha\beta = \gamma\delta$. 8

(b) Find the least possible number of imaginary roots of the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$. 8