

BSC. PART - II EXAMINATION - 2018

MATHEMATICS SUB/ GEN

Answer eight questions selecting at least one from each Group in which

Q. No. 1 is compulsory

1. Choose the correct answer of the following :-

- (a) The degree of a differential equation $\frac{d^3y}{dx^3} - \sqrt{x^2 \left(\frac{dy}{dx} \right)^{1/3}} = 0$ is
 (i) 3 (ii) 1 (iii) 6 (iv) None of these
- (b) Integrating factor of diff. equation $\frac{dy}{dx} + y \log x = 2 \cos x$ is
 (i) $\log \sin x$ (ii) $\frac{x^2}{2}$ (iii) x (iv) None of these
- (c) The equation of tangent at $(-1, 2)$ to the curve $2x^2 + 3y^2 = 14$ is given by
 (i) $x - 3y + 7 = 0$ (ii) $x - 4y + 7 = 0$ (iii) $x - 7y + 3 = 0$ (iv) None of these
- (d) If l, m, n be the direction Co-sines of a line then -
 (i) $l + m + n = 0$ (ii) $l^2 + m^2 + n^2 = 0$ (iii) $l^2 + m^2 + n^2 = 1$ (iv) None of these
- (e) The Cartesian equation of a Common catenary is -
 (i) $S = c \tan \psi$ (ii) $s = c \sec \psi$ (iii) $y = C \cosh \left(\frac{x}{c} \right)$ (iv) None of these
- (f) Order of the homogeneous function $u = \frac{x^5(x^3 - y^3)}{x^3 + y^3}$ is
 (i) 3 (ii) 2 (iii) 5 (iv) None of these
- (g) Transverse acceleration of a particle moving along a curve in a plane is
 (i) $r^2 \theta$ (ii) $\frac{d}{dt}(r^2 \theta)$ (iii) $\frac{1}{r} \frac{d}{dt}(r^2 \theta)$ (iv) None of these
- (h) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \dots$
 (i) $\log_e x$ (ii) e^a (iii) a^x (iv) None of these

GROUP - A

2. (a) Find the Standard equation of an ellipse.
 (b) Find the equation of a normal to the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1)
3. Find Co-ordinates of Vertex and focus of the parabola
 $16x^2 - 24xy + 9y^2 + 77x - 64y + 95 = 0$
4. (a) Find the equation of the plane passing through the intersection of the planes
 $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$
 (b) Find λ so that the lines $\frac{x+1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{2x-3}{3\lambda} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other
5. (a) Find the equation of a sphere which pass through the origin and makes intercepts a, b, c from Co-ordinate axes.
 (b) Find the equation of a cone whose vertex is (α, β, γ) and base
 $ax^2 + by^2 = 1, Z = 0.$

GROUP - B

6. Solve any two of the following :-

(a) $(x^2 - y^2) \frac{dy}{dx} = 2xy$ (b) $\frac{dy}{dx} + y \cot x = 2 \cos x$ (c) $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

7. Solve any two of the following :-

(a) $y = (1+p)x + ap^2$ (b) $y = 2p + p^2$ (c) $x = y - p^2$

8. (a) Prove that the system of parabola $y^2 = 4a(x+a)$ is self orthogonal

(b) Solve $x \frac{dy}{dx} = 1 + y^2$, when $x = \frac{\pi}{2}$ and $y = 1$

9. Solve any two of the following :

(a) $\frac{d^2y}{dx^2} - a^2y = e^{ax}$ (b) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$

(c) $\frac{d^2y}{dx^2} + y = \cos 2x$

GROUP - C

10. (a) Prove that a system of coplanar forces acting upon a rigid body can be reduced to a single force acting at any arbitrary point and couple whose moment is equal to the sum of the moments of the given forces about that point.

(b) Forces, PQ and R acting along the sides of a triangle formed by the lines $x=0$, $y=0$ and $x \cos \theta + y \sin \theta = p$ respectively, the area of rectangular. Find the magnitude of resultant and equation of the line of action.

11. (a) Find the intrinsic equation of a common catenary.

(b) A uniform chain of length ℓ is to be suspended from two points A and B in the same horizontal line so that the terminal tension is n times that at the lower

point then show that the span AB must be $\frac{\ell}{\sqrt{n^2-1}} \log \left\{ n + \sqrt{n^2-1} \right\}$

12. (a) A particle is moving in a plane. Find the components of its velocity along the

radius vector and perpendicular to it.

(b) The co-ordinates of a moving point at a time t are given by

$x = a(t - \sin t)$ and $y = a(1 - \cos t)$ show that it moves with constant acceleration.

13. A particle falling under gravity in a medium whose resistance varies as the velocity. find the motion of the particle.

GROUP - D

14. Examine the continuity and differentiability of the function $f(x)$ at $x=0$ where

$$f(x) = x^2 \sin \frac{1}{x}, x \neq 0$$

$$= 0, x = 0$$

15. State and prove Lagrange's mean value theorem and give its geometrical interpretation.

16. (a) If $n = \sin^{-1} \left(\frac{x^2 + y^2}{x+y} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

(b) State and prove Euler's theorem on homogeneous function in two variables.

17. (a) Discuss the necessary and sufficient conditions for $f(x,y)$ to have extreme value at (a,b) .

(b) Discuss the maximum and minimum values of $u = x^3 + y^3 - 3axy$