

BSC. PART - II EXAMINATION - 2015

MATHEMATICS SU/IV/GEN

Answer eight questions selecting at least one from each Group in which Q. No. 1 is compulsory

1. Choose the correct answer of the following :

(a) The order of a differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - x\left(\frac{dy}{dx}\right)^2 = 0$ is :

- (i) 3 (ii) 0 (iii) 2 (iv) None of these

(b) If $z = x \log y$ then $\frac{\partial^2 z}{\partial x \partial y} =$

- (i) $\frac{1}{x}$ (ii) $\frac{1}{y}$ (iii) $\log y$ (iv) e^x

(c) At any time t , for a moving particle $x = a(2t + \sin t)$ $y = a(1 - \cos t)$, the resultant acceleration is given by :

- (i) a (ii) $4a$ (iii) $3a$ (iv) None of these

(d) If e be the eccentricity of a parabola then :

- (i) $e > 1$ (ii) $e = 1$ (iii) $e < 1$ (iv) None of these

(e) The distance between the parallel planes $x + 2y - 2z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$ is :

- (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$ (iii) 2 (iv) None of these

(f) The complementary function of $(D^2 - 4D + 4)y = xe^{2x}$ is :

- (i) $(C_1 + C_2)e^{2x}$ (ii) $(C_1 + C_2)x e^{2x}$ (iii) $(C_1 + C_2)x^2 e^{2x}$ (iv) None of these

(g) If $G = 3$ $X = 1$, $Y = 2$, the equation of the line of action of the resultant is :

- (i) $2x - y + 3 = 0$ (ii) $2x + y - 3 = 0$ (iii) $x - 4y + 7 = 0$ (iv) None of these

(h) $\sin^{-1}\left(\frac{y}{x}\right)$ is a homogeneous function of degree :

- (i) 0 (ii) 4 (iii) 3 (iv) 1

GROUP - A

2. (a) Find standard equation of a parabola.

(b) Find the condition that the line $lx + my + n = 0$ may be tangent to ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

3. Find the condition that general equation of second degree in x and y may represent a parabola, an ellipse or a hyperbola.

4. (a) Find λ , so that the lines $\frac{x+1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{2x-3}{3\lambda} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other.

(b) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and

$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

5. (n) Find the equation of right circular cylinder whose radius is r and axis is the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

(b) Find the equation of cone whose vertex is (α, β, γ) and bases $ax^2 + by^2 = 1$, $z = 0$

GROUP - B

6. Solve any two of the following :

(a) $(x+y)^2 \frac{dy}{dx} = a^2$ (b) $x^2y \, dx = (x^3 + y^3)dy$ (c) $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

7. Solve any two of the following :

(a) $y = px + p - p^2$ (b) $y = 2px + p^2$ (c) $\cos^2 x \frac{dy}{dx} + y = \tan x$

8. (a) Prove that the system of parabolas $y^2 = 4a(x+a)$ is self orthogonal.

(b) Solve $(px - y)(py + x) = h^2p$.

9. Solve any two of the following :

(a) $\frac{d^2y}{dx^2} + y = \sin 2x$ (b) $\frac{dy}{dx} + y = e^{-x}$ (c) $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = xe^{2x}$

GROUP - C

10. (a) Obtain general conditions of equilibrium of system of coplaner forces acting in one plane upon a rigid body.

(b) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation to the line of action of resultant.

11. (a) Find the Cartesian equation of the common catenary.

(b) A uniform chain of length $2l$ has its extremities fastened to the fixed points at the same level and the sag in the middle is h .

Prove that the span is $\frac{L^2 - h^2}{h} \log \frac{l+h}{l-h}$

12. (a) Define S.H.M. and find its periodic time, amplitude and frequency.

(b) Find the work done in extending a light elastic string to double its length.

13. A particle is projected upwards under gravity in a resisting medium whose resistance varies as the square of the velocity. Find the motion of the particle.

GROUP - D

14. (a) Prove that a function differentiable at a point must be continuous at that point
(b) Examine the continuity and differentiability of $f(x)$ at $x = 0$

$$\text{where } f(x) = x \sin \frac{1}{x}, x \neq 0 \\ = 0, x = 0$$

15. State and prove Taylor's theorem with Lagrange's form of remainder.

16. (a) If $f(x, y) = \frac{xy}{x^2 + y^2}, x^2 + y^2 \neq 0, f(0, 0) = 0$
examine whether $f(x, y)$ is continuous at $(0, 0)$.

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x + y}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2x$

16. Discuss the Lagrange's method of undetermined multipliers.