

# BSC. PART - II EXAMINATION - 2013

## MATHEMATICS SUB/GEN

Answer eight questions in all, selecting

at least one from each Group in which Q. No. 1 is compulsory

1. Choose the correct answer of the following :

(a) Order of differential equation  $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$  is :

- (i) 1            (ii) 2            (iii) 0            (iv) 3

(b) I. F of the equation  $\frac{dy}{dx} + y \cot x = 2 \cos x$  is :

- (i)  $\log \sin x$     (ii)  $\sin x$             (iii)  $e^{\sin x}$             (iv) None of the above

(c) The value of  $\frac{1}{D^2}(e^{ax})$  is :

- (i)  $ae^{ax}$             (ii)  $\frac{e^{ax}}{a^2}$             (iii)  $\frac{e^x}{a^2}$             (iv)  $e^{ax}$

(d) Eccentricity of hyperbola  $x^2 - 9y^2 = 9$  is :

- (i)  $\frac{\sqrt{10}}{9}$             (ii)  $\frac{\sqrt{10}}{3}$             (iii)  $\frac{10}{3}$             (iv)  $\frac{5}{\sqrt{3}}$

(e) Condition that  $y = mx + c$ , touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

- (i)  $c = \sqrt{a^2m^2 + b^2}$             (ii)  $c = -\sqrt{a^2m^2 + b^2}$

- (iii)  $c = \pm \sqrt{a^2m^2 + b^2}$             (iv)  $c = \pm \sqrt{a^2m^2 + b}$

(f) The direction cosines of a line that makes equal angles with the axes is :

- (i)  $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$             (ii)  $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$

- (iii)  $\left(\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}\right)$             (iv)  $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, 0\right)$

(g) C. F. (Complementary Function) of equation  $(D^2 - 4D + 4)y = xe^{2x}$  is :

- (i)  $(C_1 + C_2)e^{2x}$     (ii)  $(C_1 + C_2x)e^x$     (iii)  $(C_1 + C_2x)e^{2x}$     (iv) None of the above

(h) The intrinsic equation of the catenary is :

- (i)  $s = c \sec \psi$     (ii)  $y = c \tan \psi$     (iii)  $s = c \tan \psi$     (iv) None of the above

### GROUP-A

2. (a) Find the standard equation of ellipse.

(b) Prove that the straight line  $y = mx + c$  touches the parabola  $y^2 = 4a(x + a)$

$$\text{if } c = ma + \frac{a}{m}$$

3. Find the condition that general equation of second degree in  $x$  and  $y$  may represent a parabola, an ellipse or a hyperbola.

4. (a) Find the equation of the plane in normal form.

(b) Find the shortest distance between the lines  $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$  and

$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

5. (a) Find the equation of right circular cylinder whose radius is  $r$  and axis is the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

- (b) Find the equation to the cone whose vertex is the origin and which passes through the curve of intersection of the plane  $lx + my + nz = p$  and the surface  $x^2 + y^2 + z^2 + 2ax + b = 0$ .

#### GROUP-B

6. Solve any two of the following :

(a)  $(x + y)^2 \frac{dy}{dx} = a^2$  (b)  $\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$  (c)  $(1 + y^2)dx = (\tan^{-1} y - x)dy$

7. Solve any two of the following :

(a)  $p^2 - p(e^x + e^{-x}) + 1 = 0$  (b)  $y = 2px + p^2$  (c)  $p^2 y = -2px + y$

8. (a) Prove that the system of confocal conics  $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$  is self orthogonal.

- (b) Solve any one of the following :

(i)  $(x + y - 1) dy = (x + y) dx$  (ii)  $\tan x \frac{dy}{dx} = 1 + y^2$  when  $x = \frac{\pi}{2}$  and  $y = 1$

9. Solve any two of the following :

(a)  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$  (b)  $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = xe^{2x}$  (c)  $\frac{d^2 y}{dx^2} + y = \cos 2x$

#### GROUP - C

10. (a) Prove that a system of coplanar forces acting on a rigid body can be reduced either to a single force or to a couple.

- (b) Three forces P, Q, R act along the sides of triangle formed by the lines  $x + y = 1$ ,  $y - x = 1$  and  $y = 2$ . Find the equation to the line of action of resultant.

11. (a) Find the Cartesian equation of the common catenary.

- (b) If T be the tension at any point P of a catenary and  $T_0$  that at lowest point C. prove that  $T^2 - T_0^2 = W^2$ , W being the weight of the arc of the catenary.

12. (a) A particle is moving in a plane curve. Find the components of its velocity along the tangent and normal to the curve.

- (b) Find the work done in extending a light string to double its length.

13. A particle falling under gravity in a medium whose resistance varies as the velocity. Find the motion of the particle.

#### GROUP - D

14. (a) Prove that a function differentiable at a point must be continuous at that point.

(b) Show that  $f(x) = x \sin \frac{1}{x}$  when  $x \neq 0$

$= 0$  when  $x = 0$

is not differentiable at  $x = 0$ .

15. State and prove Lagrange's mean value theorem. What is its geometrical interpretation.

16. (a) If  $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ .

(b) If  $f(x, y) = \frac{xy}{x^2 + y^2}$ , where  $x^2 + y^2 \neq 0$

$f(0, 0) = 0$

Examine whether  $f(x, y)$  is continuous at  $(0, 0)$  or not?

17. Discuss the Lagrange's method of undetermined multipliers.