

BSC. PART - II EXAMINATION - 2011

MATHEMATICS SUB/GEN

Answer eight questions in all, selecting at least one from each Group in which question 1 is compulsory

1. Choose the correct answers of the following :

(a) IF of the equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

- (i) e^x (ii) x^3 (iii) x (iv) $\log x$

(b) Which one is in Clairaut's form ?

- (i) $y = p^2x + f(p)$ (ii) $y = px + f(x)$ (iii) $y = px + f(y)$ (iv) $y = px + f(p)$

(c) The condition that the line $y = mx + c$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(i) $c = \pm\sqrt{b^2m^2 - a^2}$ (ii) $c = \pm\sqrt{a^2m^2 + b^2}$

(iii) $c = \pm\sqrt{a^2m^2 - b^2}$ (iv) $c = \pm\sqrt{b^2m^2 + a^2}$

(d) If $2x + 5y - 6z + 3 = 0$ be the equation of plane, then the equation of any plane parallel to the given plane is

- (i) $3x + 5y - 6z + 3 = 0$ (ii) $2x - 5y - 6z + 3 = 0$
(iii) $2x + 5y - 6z + 10 = 0$ (iv) None of these

(e) The time period of an SHM is

- (i) $\frac{\pi}{\sqrt{\mu}}$ (ii) $\frac{2\pi}{\sqrt{\mu}}$ (iii) $\frac{\pi}{2\sqrt{\mu}}$ (iv) None of these

(f) The equation of the line of action of resultant forces of the system is

- (i) $Xy + Yx = G$ (ii) $xY - yX + G = 0$
(iii) $xY - yX - G = 0$ (iv) None of these

(g) If $y = x^{n-1} \log x$, then y_n is equal to

- (i) $\ln - 1$ (ii) $\frac{\ln - 1}{x}$ (iii) \ln (iv) None of these

(h) For finding the orthogonal trajectory in differential equation, we replace $\frac{dy}{dx}$ by

- (i) $-\frac{dy}{dx}$ (ii) $\frac{dx}{dy}$ (iii) $-\frac{dx}{dy}$ (iv) None of these

GROUP-A

2. Solve any two of the following :

(i) $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ (ii) $\frac{dy}{dx} = \frac{2x+2y-1}{x+y-1}$ (iii) $xdy - ydx - \sqrt{x^2 - y^2} dx = 0$

3. (a) Solve any one of the following :

(i) $y' = 2px + p^2$, where $p = \frac{dy}{dx}$ (ii) $(D^2 + a^2)y = \cos ax$

(b) Prove that the system of parabolas $y^2 = 4a(x+a)$ is self-orthogonal.

4. Solve any two of the following :

(i) $y = px + p - p^2$ (ii) $y = px + \sqrt{a^2p^2 + b^2}$ (iii) $(px - y)(py + x) = h^2p$

5. Solve any two of the following :

(i) $\frac{d^2y}{dx^2} + 9y = \sec 3x$ (ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$

(iii) $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$

GROUP-B

6. (a) Find the standard equation of the ellipse.
 (b) Find the equation of the normal to the parabola $y^2 = 4ax$ at (x_1, y_1)
7. Find the foci and eccentricity of the conic
 $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$
8. (a) Find the equation of the plane in normal form.
 (b) Find the shortest distance between the lines.
 $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$
9. (a) Find the equation of the right circular cylinder whose radius is r and axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$
 (b) Find the equation to the cone with vertex at the origin, which passes through the curve $x^2 + y^2 + z^2 - x - 1 = 0$ and $x^2 + y^2 + z^2 + y - 2 = 0$

GROUP-C

10. (a) Prove that a system of coplanar forces acting on a rigid body can be reduced either to a single force or to a couple.
 (b) Three forces P, Q, R act along the sides of triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation to the line of action of resultant.

11. (a) Prove that $y = c \cosh \frac{x}{c}$ the Cartesian equation of the catenary.
 (b) A uniform chain of length l is to be suspended from two points A, B in the same horizontal line so that either terminal tension is n times that at the lowest point.

Prove that the span AB must be $\frac{l}{\sqrt{n^2 - 1}} \log(n + \sqrt{n^2 - 1})$

12. (a) A particle moves in a straight line OA starting from rest at A and moving with an acceleration which is always directed towards O and varies as its distance from O: find the motion. <http://www.tmbuonline.com>
 (b) Find the work done in extending a light elastic string to double its length.
13. A particle falling under gravity in a medium whose resistance varies as the velocity, find the motion of the particle.

GROUP-D

14. (a) If a function is continuous in a closed interval, prove that it is bounded in that interval.
 (b) Show that the function

$$f(x) = x \sin \frac{1}{x}, \text{ when } x \neq 0$$

$$= 0, \text{ when } x = 0,$$

is not differentiable at $x = 0$

15. State and prove Lagrange's mean value theorem. What is its geometrical interpretation?

16. (a) State and prove Euler's theorem on homogeneous function of two variables.

(b) If $f(x, y) = \frac{xy}{x^2 + y^2}$, $x^2 + y^2 \neq 0$

$$f(0, 0) = 0, \text{ examine if } f(x, y) \text{ is continuous at } (0, 0).$$

17. Discuss the Lagrange's method of undetermined multipliers.