BSC. PART - II EXAMINATION - 2010

MATHEMATICS SUB/GEN

Answer eight questions in all, selecting at least one from each Group in which questions 1 is compulsory

1. Select the correct answers from the following

(a) The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is

(i) $e^x + e^y = c$ (ii) $e^x = e^y + \frac{y^3}{3} + c$ (iii) $e^y = e^x + \frac{x^3}{3} + c$ (iv) None of these

(b) The complementary function of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}x^3$ is

(i) $(c_1 + c_2x)e^{2x}$ (ii) $c_1e^{2x} + c_2e^{3x}$ (iii) $(c_1 + c_2x)e^{3x}$ (iv) None of these

(c) If e be eccentricity of a conic section, then conic section represents a parabola if (i) $e > 1$ (ii) $e = 1$ (iii) $e < 1$ (iv) None of these

(d) If l, m, n be the direction cosines of a line, then

(i) $l + m + n = 0$ (ii) $l^2 + m^2 + n^2 = 0$

(iii) $l^2 + m^2 + n^2 = 1$ (iv) None of these

(e) The Cartesian equation of common catenary is

(i) $S = C \tan\psi$ (ii) $S = s \cosh x/c$ (iii) $S = c \sec\psi$ (iv) None of these

(f) In simple harmonic motion, frequency n is equal to

(i) $\frac{\sqrt{\pi}}{\mu}$ (ii) $\frac{2\pi}{\sqrt{\mu}}$ (iii) $\frac{\sqrt{\mu}}{2\pi}$ (iv) None of these

(g) Every differentiable function

(i) must be continuous (ii) must not be continuous

(iii) may or may not be continuous (iv) None of these

(h) If $u = x^3 + y^3 = 3axy$, then $\frac{\partial u}{\partial y}$ is

(i) $3y^2$ (ii) $3xy + y^3$ (iii) $3y^2 + 3ax$ (iv) None of the above

GROUP-A

2. Solve any two of the following

(i) $(x+y)^2 \frac{dy}{dx} = a^2$ (ii) $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ (iii) $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

3. Solve any two of the following

(i) $y = 2px + \frac{a}{p}$ (ii) $\cos^2 x\frac{dy}{dx} + y = \tan x$ (iii) $\frac{dx}{x} - \frac{dy}{y} = dx$

4. Solve any two of the following

(i) $x = \frac{1}{p} + p$ (ii) $y = 2px + 4xp^3$ (iii) $y = (1+p)x + p^2$

5. Solve any two of the following

(i) $\frac{d^2y}{dx^2} + y = e^{-x}$ (ii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x + \cos x$

(iii) $(D^3 + 2D^2 + D)y = x^2$

GROUP-B

6. (a) Find the equation of a parabola in standard form.
   (b) Find the condition for tangency of the line

   $y = mn + c$ to allipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

7. Find the equation of the tangent to the curve
   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
   at (α, β)

8. (a) Define direction cosines of a line and find the angle between the two line whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2)
   (b) Find the equation to the line through the point (1, 2, 3) parallel to the lin
   $x - y + 2z = 5,\ 3x + y + z = 6$

9. (a) Find the equation of the cone whose vertex is the point (α, β, γ) and guidin curve is the conic
   $z = 0,\ f(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
   (b) Find the equation of sphere which passes through origin and makes inte cepts a, b, c from coordinate axes

GROUP-C

10. (a) Find the equation of the line of the resultant of a system of coplanar force acting upon a rigid body.

    (b) Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$. Find the magnitude of the resultant and equation of its lines of action.

11. (a) Define common catenary and find intrinsic equation of a common catenary
    (b) A uniform chain of length 2l has its extremities fastened to two fixed points at the same level and the sag in the middle is h prove that the span is

    $$\frac{l^2 - h^2}{h} \log\left(\frac{1+h}{1-h}\right)$$

12. State and explain Hook's law. Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of its initial and final tensions.

    OR, prove that the radial acceleration $= \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$ and the transverse acceleration $= \frac{1}{r}\cdot\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)$

13. A particle of mass m is acted on by a force $m\mu\left(x + \frac{a^4}{x^2}\right)$ towards the origin. If it starts from rest at a distance a from the origin, find the time when it will arrive at the origin.

GROUP - D

14. (a) Prove that a function differentiable at a point must be continuous at that point.
    (b) Examine the differentiability of f(x) at x = 0, where

    $$f(x) = x^2 \sin\frac{1}{x},\ x \neq 0$$
    $$= 0, \quad x = 0$$

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15. State and prove Taylor's theorem with Lagrange's form of remainder.

16. (a) Examine the continuity of f(x,y) at (0,0) where

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}} \quad (x,y) \neq (0,0) \text{ and } f(0,0) = 0$$

(b) if $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

17. Discuss the necessary and sufficient condition for f(x,y) to have an exteme value at (a,b)

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