

BSC. PART - II EXAMINATION - 2009

MATHEMATICS HONOURS PAPER IV

Answer six questions in all, selecting at least one from each Group in which Q. NO.

1 is compulsory

1. Choose the correct options of the following :

(a) A partial differential equation formed by elimination function F from $Z = F(x^2 + y^2)$ is given by :

(i) $px - qy = 0$ (ii) $py - qx = 0$ (iii) $px + qy = 0$ (iv) None of these

(b) The complete integral of $f(p, q) = 0$ is :

(i) $Z = ax + \phi(b)y + c$ (ii) $Z = bx + \phi(a)y + c$

(iii) $Z = ax + \phi(a)y + c$ (iv) None of these

(c) The value of np_n is :

(i) $nP_n - P_{n-1}$ (ii) $xP_n - P_{n-1}$ (iii) $P_n - P_{n-1}$ (iv) None of these

(d) The value of $\frac{d}{dx} [x^n J_n(x)]$ is :

(i) $x^n j_{n-1}(x)$ (ii) $nx^{n-1} j_n(x)$ (iii) $nx^{n-1} J_n'(x)$ (iv) None of these

(e) $L(t \cos at)$: <http://www.tmbuonline.com>

(i) $\frac{s^2}{s^2 + a^2}$ (ii) $\frac{s^2 - a^2}{(s^2 + a^2)^2}$ (iii) $\frac{s^2 + a^2}{(s^2 - a^2)^2}$ (iv) $\frac{s}{s^2 + a^2}$

(f) Classify the following equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(i) Circle (ii) Parabolic (iii) Elliptic (iv) Hyperbolic

(g) The tangential velocity of a moving particle in a plane at P (ρ, ψ) is :

(i) ρ (ii) ψ (iii) 0 (iv) None of these

(h) The energy of a stretched elastic string is equal to :

(i) The product of the tension and extension

(ii) Half the product of the tension and the extension

(iii) The product of extension and the mean of the initial and final tensions

(iv) None of these

GROUP - A

2. Define Bessel's differential equation and find the solution of the equation.

3. Prove the following.

$$(a) P_n(x) = \frac{1}{n 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$$

$$(b) \int_{-1}^{+1} (x^2 - 1) P_{n+1} P_n dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

4. (a) Define Laplace transformation and prove that Laplace transformation is a linear transformation.

(b) Find Laplace transform of $\int_0^1 \left(\frac{1 - e^{-2x}}{x} \right) dx$

5. (a) Use convolution theorem to find. $L^{-1} \left\{ \frac{1}{p(p^2 + 4)^2} \right\}$

(b) Find $L^{-1} \left\{ \frac{1}{(s+a)(s+b)} \right\}$.

GROUP - B

6. (a) Discuss the solution of Lagrange's linear equation.

(b) Solve : $z(xp - yq) = y^2 - x^2$

7. Apply Charpit's method to find complete integral of the following :

(a) $(p^2 + q^2)y = qz$

(b) $2z + p^2 + qy + 2y^2 = 0$

8. Solve : (a) $(r + s - 6t) = Y \cos x$ (b) $p + r + s = 1$

9. Solve any two of the following by Monge's method :

(a) $1 - r \sec^2 y = 2q \tan y$

(b) $2r + te^x - (rt - s^2) = 2e^x$

(c) $3r + 4s + 1 + (rt - s^2) = 1$

GROUP - C

10. (a) Find the radial and transverse components of acceleration of a particle moving in a plane.

(b) A particle is executing S.H.M Show that it requires one sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude.

11. (a) Discuss the motion of a particle which slides down on a rough curve in a vertical plane under gravity.

(b) A particle is projected upwards under gravity (supposed constant) in a resisting medium whose resistance varies as the square of the velocity; to discuss the motion.

12. (a) Find the pedal equation of the central orbit.

(b) If a planet were suddenly stopped in its orbit, supposed circular, show that it would fall into the sun in a time which is $\sqrt{2}/8$ times the period of the planet's revolution

13. (a) Discuss motion of a particle in three dimensions

(b) In the three dimensional motion establish the expression for accelerations in spherical co-ordinates.