BSC. PART - II EXAMINATION - 2016

MATHEMATICS HONOURS PAPER III

Answer six questions in all, selecting at least one from each Group in which Q. No. 1 is compulsory

1. Choose the correct answer of the following questions :

(a) In case the double limit exists, then :

(i) The two repeated limits are necessarily equal
(ii) The two repeated limits are different
(iii) The two repeated limits may not exist
(iv) None of these

(b) A function which is contiuous is also :

(i) Uniformly continuous in closed interval [a, b]
(ii) Uniformaly continuous in open interval (a, b)
(iii) Not uniformly continous in closed interval [a, b]
(iv) None of these

(c) $R_n = \dfrac{h^n(1-\theta)^{n-1}}{\underline{|n-1}} f^n(a+\theta h)$ is known as :

(i) Lagrange's remainder
(ii) Cauchy's remainder
(iii) Scholomilch's remainder
(iv) None of these

(d) The function f(x,y) is said to have a maximum value at a point (a, b) if :

(i) $f(x, y) > f(a, b)$ (ii) $f(x, y) < f(a, b)$ (iii) $f(x, y) = f(a, b)$ (iv) None of these

(e) The function $B(m, n) =$

(i) $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$
(ii) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$
(iii) $\int_0^\infty e^{-x}x^{n-1}dx$
(iv) $\int_0^1 e^{x}x^{n-1}dx$

(f) The value of $\iint_R x^2 y\,dx\,dy$ over R, where $R : \{0 \le x \le 1; 0 \le y \le 2\}$ is :

(i) $\frac{2}{3}$ (ii) $\frac{3}{2}$ (iii) $\frac{4}{3}$ (iv) None of these

(g) Condition of equilibrium of coplanar forces acting upon a rigid body are :

(i) $X = 0,\ Y = 0$ (ii) $X = 0, G = 0$ (iii) $X = 0, Y = 0, G = 0$ (iv) None of these

(h) The Cartesion equation of a catenary contains :

(i) x & c only (ii) x & y only (iii) x, y & c only (iv) None of these

GROUP-A

2. (a) Prove that a function which is continuous in a closed interval is bounded in that interval. -52

(b) Examine the continuity and differentiablity of the function :

$$f(x) = x\sin\frac{1}{x},\ x \ne 0$$ 46

$$f(0) = 0,$$ at the point $x = 0$

3. (a) State and prove Taylor's theorem with Lagrange's remainder.

(b) Using Lagrange's Mean value theorem prove that :

$1 + x < e^x < 1 + x.e^x$, for all $x > 0$.

4. (a) Let $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ show that the repeated limits exist and are unequal but the double limit does not exist at the origin.

(b) State and prove Euler's theorem on Homogeneous function of two variables.

5. (a) State and prove Taylor's theorem for a function of two variables. 25

(b) Find the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 10$

GROUP - B

6. (a) Prove that : $\boxed{n}\ \boxed{1-n} = \dfrac{\pi}{\sin n\pi}$

(b) Evaluate : $\iint_R xy(x^2 + y^2)\,dx\,dy$ over $R\{0 \le x \le a;\ 0 \le y \le b\}$

7. (a) State and prove Cauchy's general principle of convergence.

(b) Prove that the sequence $\sqrt{2}, \sqrt{(2+\sqrt{2})}, \sqrt{\left(2+\sqrt{2+\sqrt{2}}\right)}$, ... Coverges to 2.

8. (a) State and prove Raabe's Test. Q60

(b) Discuss the convergence of the series whose nth term is $\dfrac{1}{n \log n (\log \log n)^p}$

9. (a) Prove that if a series is absolutely convergent, then it is also convergent. But the converse is not true.

(b) Show that the series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^p}$ is absolutely convergent for $p > 1$ and conditionally convergent for $0 < p \le 1$.

GROUP - C

10. (a) State and prove the necessary and sufficient condition for the equilibrium of a system of Coplanar forces acting on a rigid body.

(b) A square of side 2a is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance C apart. How that it will be in equilibrium when the inclination of one of the edges to the horizontal is either

$\dfrac{\pi}{2}$ or $\dfrac{1}{2}\sin^{-1}\left(\dfrac{a^2 - c^2}{c^2}\right)$

11. (a) State and prove the principle of virtual work, for system of Coplanar forces acting on a rigid body.

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(b) A string of length a forms the shorter diagonal of a rhombus of four uniform rods, each of length b and weight w, which are hinged together. If one of the rods be supported on a horizontal position, Prove that the tension of the string is

$$\frac{2\omega\left(2b^2-a^2\right)}{b\sqrt{4b^2-a^2}}$$ 8

12. (a) Find the intrinsic equation of a common catenary.

(b) A heavy chain of length 2 l, has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the wieght of the ring be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda}\log\left[\lambda+\sqrt{1+\lambda^2}\right]$, where $\frac{1}{\lambda}=\mu(2n+1)$ and μ is the coefficient of friction.

13. (a) Show that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the single force.

(b) Find the equation of null plane of a given point (a,b,c) referred to rectangular axis O_X, O_Y, O_Z.