BSC. PART - II EXAMINATION - 2013

MATHEMATICS HONOURS PAPER III

Answer six questions in all, selecting at least one from each Group in which Q. No. 1 is compulsory

1. Choose the correct answer of the following:

(a) The series $\sum U_n$ where $U=\dfrac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is :

(i) Divergent (ii) Convergent (iii) Oscillatory (iv) None of these

(b) The function $f(x)=\dfrac{1}{x}$ is :

(i) Uniform continuous on $0<x<1$ (ii) Continuous on $0<x<1$
(iii) Discontinuous on $0<x<1$ (iv) None of these

(c) The value of $\iint_R x^2 y\,dx\,dy$ over R, where $R:\{0\le x\le 1; 0\le y\le 2\}$ is :

(i) $2/3$ (ii) $3/2$ (iii) $\dfrac{4}{3}$ (iv) None of these

(d) If f(x) is a homogeneous function of x of degree n, then $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=nf$ is a theorem given by:

(i) Cauchy (ii) Newton (iii) Euler (iv) Liebnitz

(e) $\int_0^1 x^a(1-x)^b\,dx$ is equal to :

(i) $\dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ (ii) $\dfrac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)}$ (iii) $\dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a-b)}$ (iv) $\dfrac{\Gamma(a-1)\Gamma(b-1)}{\Gamma(a+b-2)}$

(f) If T is the tension at any point P of a catenary anf T_0 that at the lowest point then the weight W of the arc CP of the catenary is :

(i) $\sqrt{T^2-T_0^2}$ (ii) $\sqrt{T_0^2+T^2}$ (iii) $2\,TT_0$ (iv) None of these

(g) Which of the following statement is true:

(i) The virtual work done by the normal reaction is not zero
(ii) The virtual work done by the tension of an inextensible string is not zero
(iii) The virtual work done by the reaction between two bodies of the material system is zero (iv) None of these

(h) If $\vec{R}\neq 0, \vec{G}\neq 0$, then the resultant force $\vec{R}$ and the couple $\vec{G}$ can again be reduced to a single force $\vec{R}$ acting parallel to the original direction at a distance:

(i) R/G (ii) G/R (iii) G^2/R (iv) R^2/G

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GROUP-A

2. (a) Define sequential continuity. Show that the function f defined on R by:

$f(x) = 0$; x is rational

$= 1$; x is irrational

is discontinuous for every $x \in R$.

(b) Examine the continuity and differentiability at

$$x = 0 \text{ if } f(x) = |x| + x \sin\frac{1}{x}, x \neq 0$$
$$= 0 ; x = 0.$$

3. (a) State and prove Taylor's theorem with Lagrange's remainder.

(b) Show that: $1 + x < e^x < 1 + x.e^x, \forall x > 0$

4. (a) Show that the repeated limits exist at (0, 0) and are equal, but the double limit does not exist of the function: $f(x,y) = \dfrac{x^2y^2}{x^2y^2 + (x-y)^2}$

(b) If $u = f(x, y)$ is a homogeneous function of degree n, then show that:

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

5. (a) Investigate whether, $f(x, y) = x^3 - 3xy^2 + 2y^4$ has an exterme value at (0, 0)

(b) Find the maxima and minima of $x^2 + y^2 + z^2$ subject to the conditions:

$ax + by + cz = a'x + b'y + c'z = 1$

GROUP – B

6. (a) Show that : $\displaystyle\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \beta(m,n)$

(b) Evaluate $\displaystyle\iint_R [x+y]dxdy$ over the rectangle $R = [0, 1 : 0, 2]$ where $[x + y]$ denotes the greatest integer less than or equal to $x + y$.

7. (a) State and prove Cauchy's theorem on limit.

(b) Show that the sequence $\sqrt{7}, \sqrt{7+\sqrt{7}}, \sqrt{7+\sqrt{7+\sqrt{7}}}, ...$ tends to the positive root of $x^2 - x - 7 = 0$ as limit.

8. (a) State and prove Cauchy's condensation test.

(b) Test the convergence of $\displaystyle\sum_{n=2}^\infty \frac{1}{(\log n)^{\log n}}$.

9. (a) Show that if a series is absolutely convergent then it is also convergent, but the converse is not true.

(b) State and prove Leibnitz theorem on alternating series.

GROUP – C

10. (a) Explain the forces which may be omitted in forming the equation of virtual work of a system of coplanar forces acting at different points of a rigid body.

(b) Four uniform rods are joined to form a rectangle ABCD. AB is fixed in a vertical position with A uppermost and the rectangle is kept in shape by a string joining AC. Find the tension of the string.

11. (a) Find the Cartesian equation of a common catenary.

(b) A heavy chain of length 2l has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passed through A. If the weight of the string be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda}\log\left[\lambda+\sqrt{1+\lambda^2}\right]$ where $\frac{1}{\lambda}=\mu(2n+1)$ and μ is the co-efficient of friction.

12. (a) If the algebraic sum of moments of a system of coplanar forces about any three non-collinear points on their plane vanishes separately. Show that the forces of the system are in equilibrium.

(b) Three forces P, Q, R act along the sides of a triangle formed by the lines

$x + y = 1,\ y - x = 1,\ y = 2.$

Find the equation to the line of action of resultant.

13. (a) Find the equation of the central axis of a system of forces acting on a rigid body.

(b) Find the null point of the plane $x + y + z = 0$ for the force system (X, Y, Z, L, M, N)