BSC. PART - II EXAMINATION - 2010

MATHEMATICS HONOURS PAPER III

Answer six questions in all, selecting at least one from each Group, in which Q.No. 1 is compulsory and carries 20 marks The remaining questions carry 16 marks each

1. Choose the correct option(s) of the following:

(a) A function f(x,y) is said to be homogeneous of degree n if

(i) $f(x, y) = x^n f(x/y)$ (ii) $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x}$

(iii) $f(x, y) = x^n f(y/x)$ (iv) None of these

(b) If f is finitely differentiable in a closed interval [a, b] and f'(a), f'(b) are of opposite sign, then:

(i) $f'(c) = 0 \ \forall c \in [a, b]$ (ii) $f'(c) = 0$ for at least one $c \in]a, b[$

(iii) $f'(c) = 0 \ \forall c \in]a, b]$ (iv) None of these

(c) Which one of the following is not true

(i) Every constant function is continuous

(ii) Every identity function is continuous

(iii) Every polynomial function is continuous (iv) None of these

(d) The value of $\Gamma\left(\frac{1}{2}\right)$ is equal to

(i) π (ii) $\sqrt{\pi}$ (iii) 1 (iv) 2

(e) The value of $\int_0^{\pi/2}\int_0^{2a\cos\theta} r\sin\theta \, dr \, d\theta$ is

(i) a^2 (ii) $2a^2$ (iii) $2a^2/3$ (iv) $4a^2$

(f) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}$ is equal to :

(i) $1/e$ (ii) e (iii) e^2 (iv) 1

(g) is Σu_1 converges to l_1 and Σv_n converges to l_2 then $\Sigma(u_n + v_n)$ converg

(i) l_1 (ii) l_2 (iii) $l_1 + l_2$ (iv) $l_1 - l_2$

(h) Every Semi-convergent series is

(i) Convergent (ii) Absolutely convergent (iii) Divergent (iv) None of the

GROUP-A

2. (a) Prove that if a function is continuous on a closed-interval then it is bound on that interval

(b) Discuss the continuity and differentiability of the following function at x=

$$f(x) = 2 + x \text{ if } x \geq 0$$
$$= 2 - x \text{ if } x < 0$$

3. (a) State and prove Taylor's theorem for a function in a finite form with Lagrange' form of remainder

(b) Find c from the mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a} \text{ if } f(x) = x(x-1)(x-2),\ a = 0,\ b = \frac{1}{2}$$

4. (a) Show that the function $f(x, y) = xy^3/(x^2 + y^6)$; $(x,y) \neq (0, 0)$

$= 1$; $(x,4) \neq (0, 0)$

is not continuous at (0, 0)

http://www.tmbuonline.com

(b) State and prove Euler's theorem on homogeneous function of two variable

5. (a) Discuss the Lagrange's method of undetermined multipliers.

(b) Find the extreme value of the function

$$x^3 + y^3 - 6(x^2 + y^2) + 12xy - 75(x + y)$$

GROUP-B

6. (a) Prove that $\Gamma(n+1) = n\,\Gamma n,\ n > 0$.

(b) Evaluate $\int_1^2 \int_0^{y/2} y\, dy\, dx$.

7. (a) Prove that a monotonic increasing sequence bounded above tends to a limit which is its least upper bound.

(b) Examine the convergence of the sequence

$\{S_n\}$ in R, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

8. (a) State and prove Raabe's test for the convergence of the series.

(b) Apply the Cauchy's condensation test to discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{(n \log n (\log \log n)^p}$.

9. (a) State and prove Leibnits's theorem for an alternating series.

(b) Show that the series. $\sum(-1)^{n-1} \sin\left(\frac{1}{n}\right)$ is conditionally convergent.

GROUP - C

10. (a) Show that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through any given point and a single couple.

(b) Forces. P. Q. R act along the line $x = 0$, $y = 0$ and $x \cos\alpha + y \sin\alpha = p$. Find the magnitude of the resultant and the equation of its line of action.

11. (A) State and prove the converse of the principle of virtual work for a system of coplanar forces acting on a rigid body.

(b) The middle points of the opposite side of a quadrilateral are connected by light rods of length ℓ and ℓ'. If T and T' be the tensions in these rods, prove that $\frac{T}{\ell} + \frac{T'}{\ell'} = 0$

12. (a) Find the intrinsic equation of a common catenary.

(b) If T be the tension at any point P of a catenary and T_0 that at the lowest point C. Prove that $T^2 - T_0^2 = W^2$, W being the weight of the arc CP of the catenary.

13. Deduce the equation of Null lines in a plane.