BSC. PART - II EXAMINATION - 2008

MATHEMATICS HONOURS PAPER III

Answer six questions, selecting at least one from each Group in Which Q. No. 1 is compulsory

1. Choose the correct option of the following

(a) The term $R_n = \dfrac{h^n(1-\theta)^{n-1}}{\lfloor n-1} f^n(a+\theta h)$ is called

(i) Schlomilch and Roche form of remainder (ii) Cauchy's form of remainder
(iii) Lagrange's form of remainder (iv) None of these

(b) The function f defined on R^+ as $f(x) = \sin\dfrac{1}{x}\ \forall\ x > 0$ is :

(i) Uniformly continuous on R^+ (ii) Not uniformly continuous on R^+
(iii) Discontinuous on R^+ (iv) None of these

(c) If f is finitely differentiable in a closed interval $[a, b]$ and $f'(a)$, $f'(b)$ are of opposite signs, then there exists at least one point $C \in]a,b[$ such that $f'(c) = 0$ this theorem was established by

(i) Lagrange (ii) Rolle (iii) Darboux (iv) None of these

(d) $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx,\ m > 0,\ n > 0$ is known as

(i) Eulerian integral of first kind (ii) Eulerian Integral of second kind
(iii) Gamma function (iv) None of these

(e) The sequence $S_n = 1 + (-1)^n$ is

(i) Convergent (ii) Divegent (iii) Oscillatory (iv) None of these

(f) The series Σx_n is said to be conditionally convergent if

(i) Σx_n is convergent and $\Sigma|x_n|$ is not convergent

(ii) Σx_n and $\Sigma|x_n|$ both are convergent

(iii) Σx_n and $\Sigma|x_n|$ both are divergent (iv) None of these

(g) If the virtual work $= -T\,\partial l$, ∂l being the displacement, then T is known as

(i) Tension (ii) Thrust (iii) Displacement (iv) None of these

(h) The Cartesian equation of a catenary contains

(i) s and c (ii) xy and c (iii) x and y (iv) None of these

GROUP-A

2. (a) If a function is uniformly continuous on an interval then prove that it is continuous on that interval, but the converse is not necessarily true.

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(b) Examine the function f where

$$f(x) = \frac{x\left(e^{1/x} - e^{-1/x}\right)}{e^{-1/x} + e^{1/x}}, \; x \neq 0$$
$$= 0, \; x = 0$$

as regards to continuity and derivability at the origin.

3. (a) State and prove Cauchy's mean value theorem.

(b) Find C by the mean value theorem if $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \frac{1}{2}$

4. (a) Let $f : R^2 \to R$ be a function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^2 + x^2y}{x^3 + y^3}, & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Examine the continuity $f(x, y)$ at $(0, 0)$

(b) If $x = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $x\frac{\partial x}{\partial y} + y\frac{\partial x}{\partial y} = \sin 2x$

5. (a) Determine the points where a function $x^3 + y^3 - 3axy$ has a maximum or minimum.

(b) Discuss the Lagrange's method of undermined multipliers.

GROUP-B

6. (a) State and prove the recurrence formula for Gamma function

(b) Evaluate $\int_0^{\log a}\int_0^{x}\int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$.

7. (a) Prove that a monotonic decreasing sequence bounded below tends to a limit which is its greatest lower bound.

(b) Examine for convergence sequence $\{a_n\}$ where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots\ldots\ldots + \frac{1}{n+n}$$

8. (a) State and prove De-Morgan's and Bertrand's test for the convergence of an infinite series of positive terms.

(b) Prove that the series of which the nth term is $\frac{1}{n(\log n)^p}$ is convergent if $P > 1$ and divergent if $p \leq 1$.

9. (a) State and prove Leibnitz's theorem for an alternating series.

(b) Test the convergence and absolute convergence of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots\ldots\ldots p > 0$$

GROUP C

10. (a) Show that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through any given point and a single couple.

(b) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$. Find the equation to the line of action of the resultant.

11. (a) Discuss the forces which may be omitted in forming and equation of virtual work.

(b) Two equal and uniform rods AB and AC, each of length 2b, are freely joined at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b \sin^3\theta = a \cos\theta$

12. (a) Find the intrinsic equation of a common catenary

(b) Prove that the length of a heavy endless chain which will hang over a circular pulley of radius a so as to be in contact with two third of the circumference is

$$a\left\{\frac{4\pi}{3} + \frac{3}{\log\left(2+\sqrt{3}\right)}\right\}$$

13 (a) Find the resultant of any given system of forces acting at given points of a rigid body

(b) Find the condition that the straight line $\frac{x-f}{l} = \frac{y-g}{m} = \frac{z-h}{n}$ may be a null line for the same system of forces.