



2012

Time 3 hours

Full Marks 80

Candidates are required to give their answers in
their own words as far as practicable.

The questions are of equal value.

Answer any five questions in which

Q. No. 9 is compulsory.

- 1 (a) Examine for minimum and maximum values
 $\sin x + \sin y + \sin(x+y)$.
(b) Discuss the test for maxima and minima of a function at a point
- 2 (a) What do you mean by the Curvature ?
Find the radius of curvature at any point
($a \cos t : b \sin t$) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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(Turn over)

(b) If $u = x^3 \log \frac{y}{x}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$

3 (a) Solve $I = \int_{x^2}^{\frac{(x+1)}{x-1}} \frac{dx}{x+1}$

(b) Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to 3 decimals by
Trapezoidal rule with $h = 0.5, 0.25$ and
0.125

- 4 (a) Define operator Δ and E and establish the relationship between them

(b) Prove that $e^x = \left(\frac{\Delta}{E} \right) e^x = \frac{Ee^x}{\Delta^2 e^x}$, the interval
of differencing being unity.

- 5 (a) If $y = e^{at} \sin bt$, prove that $(1-x^2)y_2 - xy_1 = a^2 y$

(b) Evaluate $\int_1^2 \frac{x - \sqrt{3x-2}}{x^2-4} dx$

- 6 (a) State and prove Newton's forward difference interpolation formula

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(2)

Contd

- (b) Find the missing term in the following table:

x	y
0	1
1	3
2	9
3	—
4	81

Explain why the result differs from $3^3 = 27$.

7. (a) Evaluate $\int_{0.1}^1 \frac{1}{1+x} dx$ corrected to three decimal places

(b) Evaluate $\int_0^{2\pi} (1 + \sin x) dx$

8. (a) Show that

$$|\operatorname{Re}(2 + Z + Z^3)| \leq 4 \text{ when } |Z| \leq 1.$$

- (b) Find the principal argument $\operatorname{Arg} z$ when $z = (\sqrt{3} - i)^6$

(Objective Type Questions)

9. (a) Which of the following equation is parabolic?

(i) $f_{xy} - f_x = 0$

(ii) $f_{xx} + 2f_{xy} + f_{yy} = 0$

(iii) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$

(iv) None of these

- (b) The order of the difference equation

$y_{n+2} - 2y_{n+1} + y_n = 0$ is

(i) 1

(ii) 0

(iii) 2

(iv) None of these

- (c) $\Delta^2 e^x$ is equal to

(i) $(e - 1)^2 e^x$

(ii) $(e^x - 2)^2$

(iii) $(e - 2)^2 e^x$

(iv) None of these

- (d) The solution of $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ is

(i) $Z = f_1(y + x) + f_2(y - x)$

(ii) $Z = f_1(y + x) + f_1(y - x)$

(iii) $Z = f(x^2 - y^2)$

(iv) $Z = f(x^2 + y^2)$

(e) $\lim_{x \rightarrow 0^+} (x^n \log x) = 0$ ($x > 0$) is equal to :

- (i) 1
- (ii) 2
- (iii) 0
- (iv) None of these

(f) If ϕ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$ then $\sin \phi$ equal to

- (i) $\frac{dr}{ds}$
- (ii) $r \frac{d\theta}{ds}$
- (iii) $r \frac{d\theta}{dr}$
- (iv) None of these

(g) The radius of curvature of the curve $y = e^x$ at the point when it crosses the y-axis is

- (i) 2
- (ii) $\sqrt{2}$
- (iii) $2\sqrt{2}$
- (iv) $\frac{1}{2}\sqrt{2}$

(h) If $f(x) = f(2a - x)$ then $\int_0^{2a} f(x) dx$ is equal to

- (i) $\int_a^0 f(2a - x) dx$
- (ii) $2 \int_0^a f(x) dx$
- (iii) $-2 \int_0^a f(x) dx$
- (iv) None of these