2020

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

*Q. No. 1 carries **20** marks and remaining questions carry **16** marks each.*

*Answer **six** questions, selecting at least **two** from each Group in which Q. No. 1 is compulsory.*

1. Select the correct answer from the choices given in each of the following :

(a) If m is the mass of the particle and its acceleration is f then mf is called :

(i) Effective force

(ii) Impressed force

(iii) External force

(iv) None of these

(b) If m be the mass of an element of a rigid body and r the distance of this element from a given line then $\sum mr^2$ is :

(i) The moment of inertia of the body about the given line

(ii) The moment of inertia of the body about the given axis

(iii) The moment of inertia of the body about the centre

(iv) None of these

(c) The Lagrange's equations for a conservative holonomic dynamical system is :

(i) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial}{\partial \theta}(T - V) = 0$

(ii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \dot{\theta}} = 0$

(iii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = 0$

(iv) None of these

(d) The effective force on a particle is defined as the product of its mass m and its acceleration f. If a particle of mass m is situated at the point (x, y, z) at time t then the effective forces on this particle at this time t are :

(i) $m\frac{d^2x}{dt^2}$, $m\frac{d^2y}{dt^2}$ and $m\frac{d^2z}{dt^2}$ parallel to the axis

(ii) $m\frac{d^2x}{dt^2}$, $m\frac{d^2y}{dt^2}$ and $m\frac{d^2z}{dt^2}$ parallel to the line

(iii) $m\frac{d^2x}{dt^2}$, $m\frac{d^2y}{dt^2}$ and $m\frac{d^2z}{dt^2}$ parallel to the product of its mass

(iv) None of these

(e) The depth of the centre of pressure of a plane area immersed in a liquid is ______ than / to the depth of the C. G

(i) Less

(ii) Equal and greater

(iii) Equal

(iv) Greater

(f) Charle's law does not apply to :

(i) Vapour

(ii) Gas

(iii) Mixture of the gases

(iv) None of these

(g) The volume of a given mass of a gas under the same pressure varies directly as its :

(i) Absolute pressure

(ii) Absolute temperature and pressure

(iii) Density

(iv) None of these

(h) If p_1, ρ_1, t_1 ; p_2, ρ_2, t_2 ; p_3, ρ_3, t_3 be the corresponding values of the pressure, density and temperature of the same gas, then $\rho_1 t_1(\rho_3 p_2 - \rho_2 p_3) + \rho_2 t_2(\rho_1 p_3 - p_1 \rho_3) + \rho_3 t_3(\rho_2 p_1 - \rho_1 p_2) = K$ where K is :

(i) Greater than zero

(ii) Less than zero

(iii) Equal to zero

(iv) None of these

Group – A

2. (a) Find the moment of inertia of an ellipsoid about the axis 2a.

(b) If α, β and γ be the distances of the vertices of a triangle of mass m from any straight line in its plane, show that the moment of inertia of the triangle about this line is $\frac{m}{6}\left[\alpha^2 + \beta^2 + \gamma^2 + \beta\gamma + \gamma\alpha + \alpha\beta\right]$. Hence deduce that if h be the distance of the centre of inertia of the triangle from the line, then moment of intertia about this line is $\frac{m}{12}\left[\alpha^2 + \beta^2 + \gamma^2 + 9h^2\right]$.

3. (a) Deduce the general equation of motion of a rigid body from D'Alembert's principle.

(b) A uniform rod OA of length 2a, free to turn about its end O, revolves with uniform angular velocity W about the vertical OZ through O and is inclined at a constant angle α to OZ, show that the value of α is either zero $\cos^{-1}\left(\frac{3g}{4aw^2}\right)$.

4. (a) Find the moment of momentum about the origin of a rigid body moving in two dimensions.

(b) Find the equation of motion of rigid body under finite forces.

5. (a) State and prove the principle of conservation of linear momentum of rigid body under finite forces.

(b) An elliptic lamina is rotating about its centre on smooth horizontal table.

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If W_1, W_2, W_3 are its angular velocities when the extrimity of its major axis, its focus and the extrimity of its minor axis respectively become fixed. Prove that

$$\frac{7}{W_1} = \frac{6}{W_2} + \frac{5}{W_3}.$$

6. What do you mean by holonomic system ? State and prove Lagrange's equation of motion for a holonomic system.

Group – B

7. (a) Prove that the depth of the centre of pressure of a plane immersed in a liquid is greater than the depth of its centre of gravity.

(b) A triangle is wholly immersed in a liquid with its base in the surface. Show how to draw a horizontal line to divide it into two parts, the thrust on which are equal.

8. (a) Find the equation of surface of equi-pressure and equi-density.

(b) A portion of a sphere cut off by two planes through its centre inclined at angle $\pi/4$ is just immersed in a liquid with one face in the surface. Find the resultant thrust on the curved surface.

9. (a) Find the necessary and sufficient conditions for equilibrium of a fluid under the action of forces whose components are X, Y and Z along coordinate axis.

(b) A quadrant of circle is immersed in a liquid with a bounding radius in the surface, find the position of centre of pressure.

10. (a) Find the condition of equilibrium of a body partially or wholly immersed in a liquid and

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supported by a string attached to a point of body.

(b) Two solids are each weighed in succession in three homogeneous liquid of different densities if the weights of one are w_1, w_2, w_3 and those of other are W_1, W_2 and W_3, prove that $w_1(W_2 - W_3) + w_2(W_3 - W_1) + w_3(W_1 - W_2) = 0$.

11. (a) Find a relation among Pressure, Volume and Absolute temperature of a gas.

(b) Assuming the height of water barometer to be h, find to what depth a small inverted conical glass must be lowered so that the water may rise half up.

12. (a) Find the relation between pressure and volume in adiabatic change.

(b) If p_1, d_1, t_1 ; p_2, d_2, t_2 and p_3, d_3, t_3 are three corresponding pressures, densities and absolute temperatures of a perfect gas then prove that :

$$t_1\left(\frac{p_2}{d_2}-\frac{p_3}{d_3}\right)+t_2\left(\frac{p_3}{d_3}-\frac{p_1}{d_1}\right)+t_3\left(\frac{p_1}{d_1}-\frac{p_2}{d_2}\right)=0$$