2020

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

*Q. No. 1 carries **20** marks and the remaining questions carry **16** marks each.*

*Answer **six** questions in all, selecting atleast one from each Group, in which Q. No. 1 is compulsory.*

1. Choose the correct answer of the following questions :

(a) The order of the group S_4(symmetric group) is :

(i) 12

(ii) 18

(iii) 24

(iv) None of these

(b) The homomorphic image of an abelian group is :

(i) Abelian

(ii) Non-abelian

(iii) Isomorphic

(iv) None of these

(c) Every cyclic group of a quotient group is :

(i) Cyclic

(ii) Not cyclic

(iii) Normal group

(iv) None of these

(d) A homomorphism of a Group G to itself is called :

(i) A monomorphism

(ii) An epimorphism

(iii) An endomorphism

(iv) None of these

(e) The ring of all complex numbers is :

(i) Non commutative ring

(ii) Field

(iii) Both (i) and (ii)

(iv) None of these

(f) The necessary and sufficient condition that the non-zero element 'a' in the Euclidean ring R is a unit, is that :

(i) $d(a) \geq d(1)$

(ii) $d(a) \leq d(1)$

(iii) $d(a) = d(1)$

(iv) $d(a) \neq d(1)$

(g) A set of vectors which contains the zero vector is :

(i) Linearly dependent

(ii) Linearly independent

(iii) Both (i) and (ii)

(iv) None of these

(h) The dimension of a vector space V of all 2×2 symmetric matrices over the field F of real numbers is :

(i) 2

(ii) 3

(iii) 4

(iv) None of these

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Group – A

2. (a) Define normalizer of an element of a group and prove that normalizer N(a) of a in a group G is a subgroup of G.

   (b) Give an example to show that in a group G the normalizer of an element is not necessarily a normal subgroup of G.

3. (a) If G is a group such that $O(G) = P^n$ where P is a prime number then show that the order of centre $z > 1$.

   (b) Let G be a group, f an automorphism of G and N a normal subgroup of G then prove that f(N) is a normal subgroup of G.

4. (a) Prove that subgroup of a solvable group is solvable.

   (b) Show that a group G is solvable if and only if $G^{(k)} = \{e\}$ for some integer k.

5. State and prove Cauchy's theorem for finite abelian group.

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Group – B

6. (a) State and prove fundamental theorem on homomorphism of rings.

   (b) Show that the polynomial ring R(x) is an integral domain if R is an integral domain.

7. (a) Define skew field and show that skew field has non divisors of zero.

   (b) Prove that a field has no proper ideal.

8. (a) Prove that the ring of polynomials over a field is a Euclidean ring.

   (b) Define Ring Isomorphism. Give an example of it.

9. (a) Show that every abelian group G is a module over the ring of integers.

   (b) If A and B are two submodules of an R-module M, then prove that $A \cap B$ is also a submodule of M.

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Group – C

10. (a) Define vector space. Prove that the necessary and sufficient condition for a non-empty subset w of a vector space V(F) to be a subspace of V is a, b $\in$ F and $\alpha, \beta \in w \Rightarrow a\alpha + b\beta \in w$.

    (b) If w_1 and w_2 be two subspaces of a finite dimensional vector space V(F) then prove that :

    $\dim(w_1 + w_2) + \dim(w_1 \cap w_2) = \dim w_1 + \dim w_2$.

11. (a) If V(F) is a finite dimensional vector space then prove that any two bases of V have the same number of elements.

    (b) Prove that there exists a basis for each finite dimensional vector space.

12. Define rank and nullity of a linear transformation. Also state and prove rank and nullity theorem for linear transformation.

13. (a) Show that a linear transformation T on a finite dimensional vector space V is an isomorphism iff it is non-singular.

(b) Show that the mapping $T : V_3(R) \to V_2(R)$ defined as $T(x, y, z) = (3x - 2y + z, x - 3y - 2z)$ is a linear transformation.