2020

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

*Q. No. 1 carries **20** marks and remaining questions carry **16** marks each.*

*Answer **six** questions in **all**, selecting atleast **one** from each Group in which Q. No. 1 is compulsory.*

1. Choose the correct answer from the given alternatives :

   (a) If z is a complex number then the value of org z + org $\bar{z}$ is :

   (i) 0

   (ii) π

   (iii) $-\pi$

   (iv) None of these

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   (b) After including ideal point, the complex plane is called :

   (i) Ideal plane

   (ii) Extended complex plane

   (iii) Riemannian plane

   (iv) None of these

   (c) The value of $\frac{d^2u}{\partial z \partial z}$ for a harmonic function u(x, y) is :

   (i) 0

   (ii) 1

   (iii) – 1

   (iv) None of these

   (d) The necessary and sufficient condition for a bounded function f(x) to be R-integrable over [a, b] is that, for all partition P :

   (i) L(P, f) – U(P, f) < ε

   (ii) U(P, f) – L(P, f) < ε

   (iii) U(P, f) – L(P, f) > ε

   (iv) None of these

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(e) The value of the improper integration

$$\int_3^\infty \frac{dx}{x^2+x-2} \text{ is :}$$

(i) 3 log 5

(ii) $\frac{1}{3}\log\frac{5}{2}$

(iii) $\frac{1}{4}\log\frac{2}{5}$

(iv) None of these

(f) Any function f(x) to be expressed in the form F-series, should be assumed :

(i) Periodic

(ii) Bounded

(iii) Continuous

(iv) None of these

(g) The diameter of the empty set φ is given by :

(i) $\delta(\varphi) = -\infty$

(ii) $\delta(\varphi) = \infty$

(iii) $\delta(\varphi) = 0$

(iv) None of these

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(h) The image of a Cauchy sequence under a uniformly continuous function is a :

(i) Cauchy sequence

(ii) Oscillatory sequence

(iii) Monotomic increasing sequence

(iv) None of these

Group – A

2. (a) State and prove Darboux's theorem is Riemann Integration.

(b) If a function f is continuous on [0, 1], show that $\underset{n\to\infty}{\mathrm{Lt}} \int_0^1 \frac{n f(x)}{1+n^2x^2}\,dx = \frac{\lambda}{2} f(0)$.

3. (a) State and prove Frullani test.

(b) Test the convergancy of the improper integral

$$\int_0^\infty \frac{x\log x}{\left(1+x^2\right)^2}dx.$$

4. (a) State and prove Abel's theorem for multiplication of series.

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(b) In general it can not be said anything about the convergence property of a sequence $\{a_n, b_n\}$ if one of the sequence $\{a_n\}$ or $\{b_n\}$ diverges, even if one of them converges to zero. Justify the statement by taking suitable example.

5. (a) State and prove Young's theorem.

(b) Give and example of a function whose partial derivatives exist at the origin (0, 0) but not continuous thereat.

6. (a) If a function f is bounded periodic with period 2π and integrable on $[-\pi, \pi]$ and piecewise monotonic on $[-\pi, \pi]$ then

$$\frac{a}{2}+\sum_{n=1}^{\infty}(a_n \cos n\alpha + b_n \sin n\alpha) = \begin{bmatrix} \frac{1}{2}\{f(\alpha-)+f(\alpha+)\}, \text{ for } -\pi<\alpha<\pi \\ \frac{1}{2}\{f(\pi-)+f(-\pi+)\}, \text{ for } \alpha=\pm\pi \end{bmatrix}$$

where a_0, a_n, b_n are Fourier coefficients.

(b) Expand in a series of sines and cosines of multiple angles of x, the periodic function f with period 2π defined as

$$f(x)=\begin{Bmatrix} -1, \text{ for } -\pi<x<0 \\ 1, \text{ for } 0\le x<\pi \end{Bmatrix}.$$

Group – B

7. (a) Prove that the points z_1, z_2, z_3 are the vertices of an equilateral triangle if and only if $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.

(b) Establish the general equation of a circle is complex plane.

8. (a) Every analytic function of constant modulus is constant.

(b) Let $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = 4xy - x^3 + 3xy^3$, then find the analytic function f(z) interms of z by Milne = Thompson method.

9. (a) The cross-ratio of four points z_1, z_2, z_3, z_4 is invariant under a bilinear transformation.

(b) Find the fixed point and the normal form of the following bilinear transformation :

$$w = \frac{3z-4}{z-1}$$

Group – C

10. (a) In a metric space (x, d) :

    (i) The arbitrary union open sets is open

    (ii) Finite interaction of open sets is open

    (b) Let (x, d) be any metric space. Show that the function ρ defined by $\rho(x, y) = \dfrac{d(x,y)}{1+d(x,y)}$ $\forall x, y \in X$, is a metric on X.

11. State and prove Baire Category theorem.

12. (a) Every totally bounded metric space (X, d) is separable.

    (b) A metric space is compact if and only if it is complete and totally bounded.