AA(H-2)-Math(4)

2020

Time : 3 Hrs *Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

Q.No. 1 carries 20 marks and remaining questions carry 16 marks each.

Answer six questions in all, selecting at least one from each group in which Q.No.1 is compulsory

1. Choose the correct options of the following:

(a) The value of $P_n(1)$ is

(i) 0 (ii) 1

(iii) $\frac{1}{2}$ (iv) None of these

(b) P.I. of $(D^2 - 4)y = Sin2x$ is

(i) $\frac{1}{8}Sinx$ (ii) $-\frac{1}{8}$

(iii) $\frac{1}{4}Sin2x$ (iv) None of these

(c) The value of $\frac{d}{dx}[x^{-n}Jn(x)]$ is

(i) $-x^{-n}J_{n+1}(x)$ (ii) $-nx^{n-1}J_{n+1}(x)$

(iii) $-nx^{-n-1}J_n(x)$ (iv) none of these

P.T.O.

(d) Monge's method is applied to solve the differential equation of

(i) First degree (ii) Second degree

(iii) Third Order (iv) None of these

(e) The equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ is

(i) Wave equation (ii) Heat equation

(iii) Laplace's equation (iv) None of these

(f) The time of complete Oscillation of S.H.M. is

(i) $\frac{2\pi}{\sqrt{\mu}}$ (ii) $\frac{\sqrt{\mu}}{2\pi}$

(iii) $\frac{\pi}{\sqrt{\mu}}$ (iv) $\frac{2\pi}{\mu}$

(g) The rate of description of the sectorial area is

(i) h (ii) $h/2$

(iii) $h/3$ (iii) none of these

(h) The value of $L\{Sint\ cost\}$ is

(i) $\frac{1}{P^2+4}, P>0$ (ii) $\frac{1}{P^2+2}, P>0$

(iii) $\frac{1}{P^2+3}, P>0$ (iv) None of these

Group-A

2. Define Bessel's differential equation and find the solution of the equation.

3. (a) Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$

   (b) Prove that $'P_n = xP_n' - P_{n-1}'$

4. Prove that Laplace transformation of the following:

   (a) $Cos^2(2t)$ (b) $Cos\,h(at)$

5. State and Prove First and Second shifting theorem in Laplace transformation.

Group-B

6. Solve any two of the following:

   (a) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

   (b) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

   (c) $q(p^2z + q^2) = 4$

7. Using Charpit's method solve any two of the following:

   (a) $Px + qy = pq$

   (b) $z^2(p^2z^2 + q^2) = 1$

   (c) $pxy + pq + qy = yz$

8. Solve any two of the following:

   (a) $r - 2s + t = Sin(2x + 3y)$

   (b) $(D^3 - 4D^2D^1 + 4DD'^2)z = 4Sin(2x + y)$

   (c) $r + s - 6t = y\cos x$

9. Solve any two of the following differential equation by Monge's method:

   (a) $(q+1)s = (p+1)t$

   (b) $r - t\cos^2 x + p\tan x = 0$

   (c) $2r + te^x - (rt - s^2) = 2e^x$

Group-C

10. (a) Find the radial and transverse components of velocity of a particle moving in a plane.

    (b) Find the work done in extending a elastic string to double its length.

11. State and Prove Kepler's law of planetary motion.

12. (a) Find the differential equation of central orbit in polar form.

    (b) A particle moves with a central acceleration $\frac{\pi}{(distance)^2}$: it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is $Sin^{-1}\left[\frac{\pi}{VR\left(V^2 - \frac{2\pi}{R}\right)^{1/2}}\right]$.

13. Discuss motion of particle in three dimension, Establish the expression for acceleration in Cartesian co-ordinates in three dimensional space.

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