2019

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Q. No. 1 carries **20** marks and the remaining questions carry **16** marks each. Answer **six** questions in all, selecting at least **one** from each Group, in which Q. No. 1 is compulsory.*

1. Choose the correct answer of the following :

(a) If 0 (Avt G) > 1, then 0(G) is :

(i) Greater than 2

(ii) Less than 2

(iii) Equal to 2

(iv) None of these

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(b) If G is a group of order 3^5, then the order of Z(G) is :

(i) Greater than 1

(ii) Greater than 3

(iii) Greater than 5

(iv) None of these

(c) A group of order 51 is :

(i) Non-solvable

(ii) Solvable

(iii) Non-cyclic

(iv) None of these

(d) The ring R = {0, 1} mod 2 has characteristic :

(i) Zero

(ii) One

(iii) Two

(iv) None of these

(e) In a principal ideal domain every non-zero prime ideal is :

(i) Minimal

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(ii) Maximal

(iii) Neither Minimal nor Maximal

(iv) None of these

(f) If $W_1 = \{(a, 0) : a \in R\}$ $W_2 = \{(0, b) : b \in R\}$ are sub-spaces of R^2, then $W_1 \cup W_2$ is :

(i) Subspace of R^2

(ii) Not a subspace of R^2

(iii) Subspace of R^3

(iv) None of these

(g) Let $S = \{(1, 4), (0, 3)\}$ be a subset of $R^2(R)$, then which one of the following is true ?

(i) $(2, 3) \notin L(S)$

(ii) $(2, 3) \in L(S)$

(iii) $(2, 3) \notin R^2(R)$

(iv) None of these

(h) Matrix of linear transformation $T : R^2 \to R^2$ defined by $T(x_1, x_2) = (-x_2, x_1)$ with respect to standard basis $\{(1, 0), (0, 1)\}$ is :

(i) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

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(ii) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(iv) None of these

Group – A

2. (a) Prove that the centre Z of a group G is a normal subgroup of G.

   (b) Let Z be the centre of a group G. Then prove that G is abelian if $\frac{G}{Z}$ is cyclic.

3. Prove that the set I(G) of all inner automorphism of a group G is a normal subgroup of the group of its automorphism is isomorphic to quotient group $\frac{G}{Z}$ of G where Z is the centre of G.

4. (a) Define solvable group. Prove that every abelian group is solvable.

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(b) If N is normal subgroup of G such that both N and G/N are solvable, then prove that G is solvable.

5. State and prove Sylow first theorem.

Group – B

6. (a) Prove that every Euclidean domain is a principal ideal domain.

(b) Prove that the set R[x] of all polynomial over a field is not a field.

7. (a) Prove that every Euclidean ring is a principal ideal ring.

(b) If S is an ideal of ring R, then show that $R/S = \{S + a \, ; \, a \in R\}$ forms a ring under suitable operation.

8. State and prove unique factorization theorem for polynomials over a field.

9. (a) Write down the general properties of module and prove them.

(b) Show that every ideals in ring R is module over R.

Group – C

10. (a) Prove that union of two subspaces of a vector space V(F) is not necessarily a subspace of V(F).

(b) If W_1 and W_2 are sub-spaces over a vector space V(F), then prove that $W_1 + W_2$ is also a subspace of V(F).

11. (a) Prove that linearly independent subset of a finitely generated vector space V(F) is either basis of V or can be extended to form of basis of V.

(b) Are the three vectors (1, 1, – 1), (2, – 3, 5) and (– 2, 1, 4) of R^3 linearly independent ? Justify your assertion.

12. (a) Prove that the set of all linear transformations from U(F) to V(F) is an abelian group with respect to two linear transformation.

(b) Let T : U(F) → V(F) be a linear transformation, then prove that Range R(T) of T is subspace of V(F).

13. Let U(F) and V(F) be two vector spaces show that the set of all linear transformation from U(F) to V(F) forms a vector space under suitable operation.