2018

Time : 3Hrs *Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

Q. No. 1 carries 20 marks and the remaining questions carry 16 marks each.

Answer six questions in all, selecting at least one from each group, in which Q. No. 1 is compulsory.

1. Choose the correct answer of the following:

(a) The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.........$ is

(i) Semi convergent

(ii) Absolutely convergent

(iii) Divergent

(iv) None

(b) If the sequence $\{U_n\}$ is bounded and $\sum V_n$ converges absolutely, then $\sum U_n V_n$

(i) Converges

(ii) Diverges

(iii) Converges absolutely

(iv) None

(c) The product of two given series $\sum a_n$ and $\sum b_n$ is

(i) $\sum_{k=1}^{n} a_k b_{n-k+1}$

(ii) $\sum_{k=1}^{n} a_{k-n} b_k$.

(iii) $\sum_{k=1}^{n} a_k b_{n-k}$

(iv) None

(d) Fourier Coefficient of an even function f(x)

(i) $a_n = 0$

(ii) $a_n = b_n$

(iii) $b_n = 0$

(iv) None

(e) The conjugate of $(1+i)^2$ is

(i) $(1+i)^{-1}$

(ii) $(1+i)^{-2}$

(iii) $-2i$

(iv) None

(f) An equation of the form $z\bar{z}+b\bar{z}+\bar{b}z+c=0$ will represent a

(i) Straight line

(ii) Circle

(iii) Triangle

(iv) None

(g) Let (x, d) be a metric space and let $A \leq X$, then the derived set D(A) is

(i) Open

(ii) Closed

(iii) Both (i) and (ii)

(iv) None

(h) If A is a subset of a metric space (x, d), then relation between interior points and closure, where 'c' stands for the complement is given by

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(i) $(\bar{A})^c = (A^c)^0$

(ii) $(\bar{A})^c = (A^0)^c$

(iii) $(\bar{A})^c \neq (A^c)^0$

(iv) None

Group-A

2. (a) Prove that every continuous function is Riemann integrable.

(b) If a bounded function f(x) is R-integrable on [a,b], show that |f(x)| is bounded and R-integrable on [a, b] and that

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$

3. (a) State and prove Abel's Test for improper integral.

(b) Prove that $\int_0^\infty \frac{\sin x}{x}$ is convergent.

4. (a) State and prove Merten's theorem for multiplication of series. 304

(b) Test the convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{h}\right)\frac{\sin n\theta}{n}$

5. (a) State and prove Young's theorem.

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(b) Discuss the differentiability at the origin of the function.

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \; ; \; (x, y) \neq (0,0)$$

$$= 0 \qquad ,\text{otherwise}.$$

6. (a) State and prove Riemann localization theorem.

(b) Find the Fourier Series of the function

$f(x) = |\sin x|$ in $-\pi \leq x \leq \pi$.

Group B

7. (a) Prove that modulus of sum or difference of two complex numbers is always less than or equal to the sum of their moduli.

(b) If Z_1, Z_2, Z_3 are the vertices of an isosceles triangle right angled at the vertex Z_2, prove that

$$Z_1^2 + 2Z_2^2 + Z_3^2 = 2Z_2(Z_1 + Z_3).$$

8. (a) State and prove the polar form of Cauchy Riemann equations.

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(b) Show that the function $e^{-z^{-4}}, (z \neq 0), f(0) = 0$ is not analytic at $z = 0$, although the Cauchy Riemann equations are satisfied at the point. How would you explain it?

(a) Prove that the cross-ratio of four points is invariant under a bilinear transformation.

(b) Show that every bilinear transformation with two finite fixed points α, β can be put in the form

$$\frac{w - \alpha}{w - \beta} = \lambda\left(\frac{z - \alpha}{z - \beta}\right).$$

Group-C

10. (a) Prove that in a metric space every open sphere is an open set.

(b) Let (x, d) be a metric space, then prove that the function d^* defined by

$$d^*(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \; * x, y \in x \text{ is a metric on } x.$$

11. (a) Let X be a metric space and $A, B \leq X$, then prove that

(i) $A \subseteq B \Rightarrow D(A) \subseteq D(B)$

(ii) $D(A \cup B) \Rightarrow D(A) \subseteq D(B)$

(b) Let (x, d_1) and (y, d_2) be two metric spaces then the mapping $f : x \to y$ is continuous at $a \in x$ iff for every sequence $\{x_n\}$ in X converging to a, the sequence $\{f(x_n)\}$ in Y converging to f(a).

12. State and prove Baire's Category theorem.

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