AA(H-2)-Math(3)

2020

Time : 3 Hrs *Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

Q.No. 1 carries 20 marks and remaining questions carry 16 marks each.

Answer six questions in all, selecting at least one from each group in which Q.No. 1 is compulsory.

1. Choose the correct answer of the following:

(a) For the existence of limit of any function at any point, the function

(i) May be or may not be defined at the point

(ii) Must be defined at that point.

(iii) Never be defined at that point.

(iv) None of these

(b) Any sequence of irrational numbers may be convergent

(i) to a rational point

(ii) to an irrational point

(iii) both of (i) & (ii)

(iv) None of these

(c) Lagrange's Mean value theorem is a particular case of

P.T.O.

(i) Taylor's theorem

(ii) Cauchy-Mean Value theorem

(iii) Darboux's theorem

(iv) None of these

(d) If $\underset{(x,y)\to(a,b)}{Lt} f(x,y) = \ell(exists)$, $\underset{x\to a}{Lt} f(x,y)$ exists for each constant value of y in the $\frac{nhd}{..}$ of y=b and $\underset{y\to b}{Lt} f(x,y)$ exists for each constant value of x in the neighbourhood of $x=a$ then $\underset{x\to a}{Lt}\ \underset{y\to b}{Lt} f(x,y) = \underset{y\to b}{Lt}\ \underset{x\to a}{Lt} f(x,y) = e,$

is the statement of

(i) Double limit theorem

(ii) Repeated limit theorem

(iii) Moore-Osgood theorem

(iv) Existence of limit theorem

(e) Beta function $B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is defined for

(i) m>1 and n>1

(ii) $m>0$ & $n>0$

(iii) $m<0$, $n<0$

(iv) m<0 & n>0

(f) The value of $\int_0^1\int_0^1 \frac{1}{(1+x^2)(1+y^2)}dxdy$ is equal to

(i) $\frac{\pi^2}{2}$

(ii) $\frac{\pi^2}{4}$

(iii) $\frac{\pi^2}{8}$

(iv) $\frac{\pi^2}{16}$

(g) If the algebraic sum of the moments of the system of coplanar forces about a point vanishes then the resultant of the system of forces

(i) passes through the point

(ii) does not pass through the point

(iii) May or may not pass through the point

(iv) None of these

(h) The string of the common catenary is

(i) inextensible

(ii) perfectly flexible

(iii) perfectly flexible and inextensible

(iv) flexible and inextensible

Group-A

2. (a) If any function $f(x)$ is continuous is [a,b] then it attains its founds at least once in [a,b]. Prove it.

(b) If $f(x)$ is defined on $]0,1[$ by

$f(x) = 0$, when x is irrational

$f(x) = \frac{1}{q}$ when $x = \frac{p}{q}$ a rational for $p, q \in N$ in their lowest term, then show that f is continuous for each irrational point and discontinuous at each rational point in $]0, 1[$.

3. (a) State and prove Rolle's theorem.

(b) If $f(x+h)$ admits of Taylor's expansion with Lagrange's form of remainder after n terms viz $Rn = \frac{h^n}{\lfloor n} f^n(a + Qh)$ and $f^{n+1}(x)$ is continuous in a neighbourhood of x containing $x + h$ and $f^{n+1}_{(x)} \neq 0$, show that $Q \to \frac{1}{n+1}$ as $h \to 0$.

4. (a) Discuss the criteria for determination of maxima or minima of a function of two variables.

(b) Find the maxima and minima of the function $x^3 + y^3 - 3axy, x \neq 0, y \neq 0$.

5. (a) State and prove Euler's theorem for the homogeneous function of two variables.

(b) Using $\in -\delta$ definition of continuity examine the continuity of $f(x,y)$ at $(O,0)$ when $f(x,y) = \frac{xy(x^2 - y^2)}{(x^2 + y^2)}$ when $(x,y) \neq (0,0)$ & $f(0,0) = 0$.

Group-B

6. (a) State and prove the Cauchy's general principle of convergence of a sequence.

(b) If $a_1 < a_2$ are arbitrary real numbers and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$ for n>2, show that $\{x_n\}$ is convergent and find $\lim_{n \to \infty} x_n$.

7. (a) State and prove Cauchy's condensation test.

(b) Examine the convergence of the series

$$1 + \frac{1}{2} \cdot \frac{x^2}{4} + \frac{1.3.5}{2.4.6} \cdot \frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10} \cdot \frac{x^6}{12} + \ldots\ldots$$

8. (a) State and prove Leibnitz's theorem for an alternating series.

(b) Test the convergency of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$.

9. (a) Express $\int_0^{\pi/2} \cos^m\theta \ \sin^n\theta \ d\theta$ in terms of Gamma function.

(b) Evaluate $\iiint_D \left(x^2 + x^2\right) \ dxdydz$ where

$$D = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$$

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Group-C

10. (a) To find the equation of the line of action of the resultant of a system of coplanar forces acting at different points of a rigid body.

(b) A solid cone of height h and Semi-vertical angle α is placed with its base against a smooth vertical wall and is supported by a string attached to its vertex and to a point in the wall. Show that the greatest possible length of the string is $h\sqrt{1 + \frac{16}{9} \tan^2 \alpha}$

11. (a) State and prove the principle of virtual work for system of coplanar forces acting on different points of a rigid body.

(b) A heavy elastic string whose natural length is $2\pi a$ placed round a smooth cone whose axis is vertical and whose-semi vertical angle is α. If ω be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of the circle whose radius is $a\left(1 + \frac{\omega}{2\pi\lambda} \cos \alpha\right)$.

12. (a) For a catenary with parameter C, prove that Hvat. $x = c \log(\sec\Psi + \tan\Psi)$, where ψ is the angle of tangency of the catenary at (x, y).

(b) Prove that the length of a heavy endless chain which will hang over a circular pulley of Radius a so as to be in contact with two-third of circumference is

$$a\left(\frac{4\pi}{3}+\frac{3}{\log\left(2+\sqrt{3}\right)}\right).$$

13. (a) To prove that for a system in equilibrium under conservative forces, the potential energy is minimum for stable and maximum for unstable equilibrium.

(b) To determine the condition in order that a given system of forcess should compound into a single force.

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