AA(H-1)–M(2)

2018

Time : 3 hours *Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

Q.No.1 Carries 20 marks and remaining questions carry 16 marks each.

Answer Six questions in all, Selecting at last one from each group in which Q.No.1 is compulsory.

1. Choose the correct answer of the following:–

(a) If $y = \cos(ax + b)$ then y_{n+1} is equal to

(i) $a^n \sin\left(\frac{n\pi}{2} + b\right)$

(ii) $a^n \cos\left(\frac{n\pi}{2} + ax + b\right)$

(iii) $a^n \sin(n\pi + ax + b)$

(iv) none of these

(b) $\lim_{x \to c} \frac{x}{\sin x} = \ldots\ldots$

(i) 1

(ii) 0

(ii) ∞

(iv) none of these

(c) Pedal equation of the parabola $y^2 = 4ax$ w.r.t. its focus is

(i) $p = ar$

(ii) $p^2 = ar$

(iii) $r = ap$

(iv) none of these

(d) The degree of differential equation

$$\left(1 + \frac{d^2y}{dx^2}\right)^{3/2} = 2\left(\frac{dy}{dx}\right)^5 \text{ is}$$

(i) 3

(ii) 10

(iii) 6

(iv) none of these

(e) $\int_0^\infty \frac{\sin(\tan^{-1} x)}{1+x^2} dx = \ldots\ldots$

(i) $\frac{\pi}{2}$

(ii) $\frac{\pi}{4}$

(iii) 3

(iv) none of these

Group – A

2. (a) Discuss the continuity of the following function at x = 0,

$$f(x) = x^2\sin\frac{1}{x} + x\cos\frac{1}{x} \text{ when } x \neq 0$$

$$f(x) = 0$$

(b) Show that the function defined by

$$f(x) = x^2\sin\frac{1}{x};\ x \neq 0$$

$$= 0;\ x = 0$$

is differentiable for every value of x but the derivative is not continuous at x = 0.

3. (a) If $y = \frac{1}{x^2 + a^2}$ then show that.

$$y_n = \frac{(-1)^n \lfloor n}{a^{n+2}} \sin^{n+1}\theta.\sin(n+1)\theta$$

where $\tan\theta = \frac{a}{x}$.

(b) Apply Maclaurin's Theorem to obtain the expansion of secx.

(iii) 1

(iv) none of these

(f) If θ be angle between vectors $\vec{a}$ and $\vec{b}$ with the condition $|\vec{a}\times\vec{b}| = |\vec{a}.\vec{b}|$, then θ is equal to

(i) 0°

(ii) 45°

(iii) 180°

(iv) none of these

(g) The eccentricity of the ellipse.

$9x^2 + 25y^2 = 225$ is

(i) $\pm\frac{4}{5}$

(ii) $\frac{4}{5}$

(iii) $\frac{2}{3}$

(iv) none of these

(h) The length of the normal drawn from the origin to the plane.

$x - 4y + 8z - 27 = 0$ is given by

(i) 27

(ii) 9

4. (a) Integrate $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

(b) Evaluate $\int_0^{\pi/2} \cos^h x . \cos hx \, dx$

5. (a) Find the area of the loop of the curve.

$y^2 = x(x-1)^2$

(b) Find the length of the arc of the curve.

$\frac{2a}{r} = 1 + \cos\theta$ from $\theta = 0$ to $\theta = \pi/2$

Group – B

6. Solve any two of the following:-

(a) $\frac{dy}{dx} + \frac{xy}{1-x^2} = x\sqrt{y}$

(b) $p(p+x) = y(x+y)$

(c) $x^2\left(y - x\frac{dy}{dx}\right) = y\left(\frac{dy}{dx}\right)^2$

7. (a) Prove that the system of confocal conics

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$$

is self orthogonal.

(b) Solve $\frac{d^2y}{dx^2} - Z\frac{dy}{dx} + y = x^2e^{3x}$

8. (a) Prove that

$$[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}+\vec{b}\ \vec{c}]$$

(b) Prove that

$$(\vec{b}\times\vec{c}).(\vec{a}\times\nabla)+(\vec{c}\times\nabla).(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=0$$

9. (a) Prove that http://www.tmbuonline.com curve:

$$(\vec{a}\times\vec{b}) = (\vec{b}.\nabla)\vec{a} - (\vec{a}\times\nabla)\vec{b} + \vec{a}\ div\vec{b} - \vec{b})\ div\ \vec{a}$$

(b) If $\vec{F} = (x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}$, then, show that $\vec{F}$ curve $\vec{F} = 0$

Group – C

10. (a) Find the standard equation of an ellipse.

(b) If the chord is normal to the parabola at one end and subtends a right angle at the vertex, prove that it is inclined at an angle $\tan^{-1}(\sqrt{2})$ to the axis.

11. (a) Find the equation of the tangent to curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ at the point (x_1, y_1).

(b) Determine the foci and directrices of the conic

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0.$$

12. (a) Find the angle between the lines whose direction cosines (l, m, n) satisfy the equations $l + m + n = 0$ and $2lm + 2ln - mn = 0$.

(b) Find the magnitude and the equation of the shortest distance between two given straight lines.

13. (a) Find the equation of the cone whose vertex is the point (α, β, γ) and base is the conic.

$f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and $z = 0$

(b) Find the equation of e sphere which touches the sphere $x^2 + y^2 + z^2 = 9$ at the point $(1, 2, 2)$ and passes through the origin.

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