AA(H-I) — Math (2)

2023

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

*Answer **six** questions, selecting at least **one** from each Group in which Q. No. 1 is compulsory.*

Q. No. 1 carries 20 marks and remaining questions carry 16 marks each.

1. Choose the correct answer of the following :

(a) If $y = xe^{ax}$, then y_n is equal to :

(i) $na^{n-1}e^{ax}$

(ii) $a^n e^{ax}$

(iii) $a^n e^{ax} + na^{n-1}e^{ax}$

(iv) None of these

(b) The function $f(x) = |x-1| + |x+2|$ is

(i) Differentiable at 1 and –2

(ii) Not differentiable at 1 and –2

(iii) Not continuous at 1 and –2

(iv) None of these

(c) The integral $\int_1^{\sqrt{e}} x \ln x\,dx$ is equal to :

(i) $\frac{1}{4}$

(ii) $\frac{1}{2}$

(iii) $\frac{1}{3}$

(iv) None of these

(d) The order and degree of the differential equation $\left\{1+\frac{dy}{dx}\right\}^{2/3} = \frac{d^2y}{dx^2}$ are

(i) 2, 1

(ii) 1, 2

(iii) 2, 3

(iv) None of these

(e) The P. I. of the differential equation $(D^2 - 4D + 4)y = xe^{2x}$ is

(i) $\frac{x^3}{3}e^{2x}$

(ii) $\frac{x^3}{6}e^{2x}$

(iii) x^3e^{2x}

(iv) None of these

(f) The angle between the normals to the surface $xy = z^2$ at the points (1, 4, 2) and (−3, −3, 3) is

(i) $\mathrm{Cos}^{-1}\left(\frac{1}{\sqrt{22}}\right)$

(ii) $\mathrm{Sin}^{-1}\left(\frac{1}{\sqrt{22}}\right)$

(iii) $\mathrm{Cos}^{-1}\left(\frac{1}{\sqrt{11}}\right)$

(iv) None of these

(g) The angle between the tangents at the extremities of any focal chord is

(i) $\frac{\pi}{4}$

(ii) $\frac{\pi}{2}$

(iii) $\frac{\pi}{3}$

(iv) None of these

(h) The equation of plane passing through the points (0, 0, a) (0, b, 0) and (c, 0, 0) is

(i) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(ii) $\frac{x}{c}+\frac{y}{b}+\frac{z}{a}=1$

(iii) $cx + by + az = 1$

(iv) None of these

Group – A

2. (a) Examine the differentiability at $x = 0$ of

$$f(x) = \begin{cases} e^{\frac{-1}{x^2}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

(b) If a function f defined in a closed interval [a, b] possesses a finite derivative at a point $c \in (a, b)$ then prove that f is continuous at c.

3 (a) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that

$$(x^2 - 1)\, y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2) y_n = 0$$

(b) Expand Secx in ascending powers of x up to x^4

4. (a) Evaluate any two of the following :

(i) $\int_0^1 \tan^{-1} x\, dx$

(ii) $\int_0^1 \frac{\ln(1+x)}{1+x^2}\, dx$

(iii) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

5. (a) Find the area of the loop of the curve

$$3y^2 = x^2 (3 - x)$$

(b) Find the entire length of the cardiode

$$r = a\,(1 + \cos\theta).$$

Group – B

6. Solve any two of the following

(i) $x\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(ii) $(2x + y)\, dx - (4x + 2y^{-1})\, dy = 0$

(iii) $\frac{dy}{dx} + \frac{1}{x} \sin 2y = x^3 \cos^2 y$

7. (a) Prove that the system of parabolas $y^2 = 4a\,(x + a)$ is self orthogonal.

(b) Solve $(D^2 - 2D + 1)y = e^x + xe^{2x} - 2\sin x$.

8. (a) Prove that $[\vec{a} + \vec{b},\ \vec{b} + \vec{c},\ \vec{c} + \vec{a}] = 2\,[\vec{a}, \vec{b}, \vec{c}]$.

(b) If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors then prove that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$.

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9. (a) Prove that $\text{Div}(\vec{a} \times \vec{b}) = \vec{b} \cdot (\text{curl}\ \vec{a}) - \vec{a} \cdot (\text{Curl}\ \vec{b})$

(b) Evaluate $\nabla^2\left(\frac{1}{r}\right)$ where

$r = |\vec{r}| = \sqrt{(x^2 + y^2 + z^2)}$

Group – C

10. (a) Find the standard equation of Hyperbola.

(b) Find the condition that the line $y = mx + c$ will touch the parabola $y^2 = 4ax$

11 (a) Find the polar equation of Conic when the focus is the pole.

(b) Find the equation of tangent to the general conic.

12. (a) If α, β, γ are the direction cosines of a line then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

(b) Find the equation of plane in the normal form.

13 (a) Find the equation of the sphere on the join of (2, -3, 1) and (3, -1, 2) as diameter.

(b) Prove that from a given point, six normals can be drawn to the central conicoid

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