

# 2020

Time : 3 Hrs

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

Q.No.1 carries 20 marks and remaining questions carry 16 marks each.

Answer six questions, selecting at least one from each Group, in which Q.No.1 is compulsory.

1. Choose the correct answer from option given below.

(a) Let  $A = \{1,2,3\}$  and  $B = \{1,2,3,4,5\}$

The function  $g = \{(1,1), (2,3), (3,3)\}$  is

- (i) one to one
- (ii) onto
- (iii) one to one and onto
- (iv) none of these

(b) If  $A = \{1,2,3\}$  and  $R = \{(1,1), (1,2), (2,2), (2,3), (3,3)\}$  is a relation of A then R is

- (i) Symmetric
- (ii) Reflexive

- (iii) Transitive
- (iv) None of these

(c) The set of all positive irrational numbers under the operation of ordinary multiplication is

- (i) an abelian group
- (ii) Not a group
- (iii) Non-abelian group
- (iv) none of these

(d) The equation  $x^6 - 4x^2 - 2x + 1 = 0$  will have

- (i) At least two imaginary roots
- (ii) At least one imaginary root
- (iii) No imaginary roots
- (iv) None of these

(e)  $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} - \vec{b})\vec{c} = \dots\dots$

(i)  $\vec{a}(\vec{b} \times \vec{c})$

(ii)  $(\vec{a} \times \vec{b})\vec{c}$

(iii)  $\vec{a} \times (\vec{b} \times \vec{c})$

(iv)  $\vec{a} \times (\vec{b} \times \vec{c})$

P.T.O.

**Group-A**

(f)  $(\cos \theta - i \sin \theta)^{-n} = \dots\dots$

(i)  $\cos n \theta + i \sin (n-1) \theta$

(ii)  $\cos n \theta - i \sin n \theta$

(iii)  $\cos n \theta + i \sin n \theta$

(iv) none of these

(g) If  $\begin{bmatrix} 3x-4y & 8 \\ -5 & 4x+y \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -5 & 3 \end{bmatrix}$  then  $(x,y) = \dots\dots\dots$

(i) (1,1)

(ii) (-1,-1)

(iii) (1,-1)

(iv) none of these

(h)  $\int_0^{\pi/2} \sin^6 X \, dx$  is equal to

(i)  $\frac{3\pi}{17}$

(ii)  $\frac{5\pi}{18}$

(iii)  $\frac{5}{9}$

(iv) none of these

2. (a) If X and Y are two non empty sets and  $f$  is a mapping of X into Y then for any two subsets A and B of X, prove that  $f(A \cup B) = f(A) \cup f(B)$ .

(b) If  $F : R \rightarrow R, f(x) = x^2, g : R \rightarrow R, g(x) = x + 3$  then prove that  $(g \circ f)(2) \neq (f \circ g)(2)$

3. State and prove fundamental theorem on equivalence relation.

4. (a) Prove that in equation with real coefficients imaginary roots occur in conjugate pairs.

(b) Solve the equation  $2x^3 - 15x^2 + 37x - 30 = 0$  whose roots are in A.P.

5. Solve  $x^3 - 7x + 6 = 0$  by Cardon's method.

**Group-B**

6. (a) Let G be a group, prove that  $(ab)^{-1} = b^{-1}a^{-1} \forall a, b \in G$

(b) Prove that a necessary and sufficient condition for a non-empty subset H of a group G to be a sub group is that  $a \in H, b \in H \Rightarrow ab^{-1} \in H$  where  $b^{-1}$  is the inverse of b in G. <https://www.tmbuonline.com>

7. (a) State and prove Langrange's theorem for a finite group.

(b) Write down all the permutations on the three symbol a, b, c which of three permutation are even?

8. (a) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew-symmetric matrix.

(b) Find the inverse of the matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

9. (a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(b) If A and B are two non-singular matrices of same order then prove that  $(AB)^{-1} = B^{-1}A^{-1}$

### Group-C

10. State and prove De-Moivre's theorem.

11. (a) Express  $\sin^{-1}(x+iy)$  in the form of  $A+iB$  where A and B are reals.

(b) Expand  $\cos x$  in ascending powers of  $x$ .

12. (a) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

(b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$

13. (a) Prove that  $\frac{d}{dt}(\vec{u} \pm \vec{v}) = \frac{d\vec{u}}{dt} \pm \frac{d\vec{v}}{dt}$  where  $\vec{u}$  and  $\vec{v}$  are two differentiable functions of the scalar  $t$ .

(b) If  $\vec{a}$  is a constant vector, then prove that  $\text{div } \vec{a} = 0$ .

### Group-D

14. (a) Prove that every differentiable function is continuous but the converse is not true.

(b) If  $f(x) = x \sin \frac{1}{x}$ ,  $x \neq 0$

$f(0) = 0$  then show that  $f(x)$  is continuous at  $x = 0$

15. (a) State and prove Taylor's theorem.

(b) Find  $Y_n$  if  $y = \frac{1}{x^2 + a^2}$

16. Integrate any two of the following:

(a)  $\int \frac{dx}{(x-1)\sqrt{x+2}}$

(b)  $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$

(c)  $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$

(d)  $\int_0^1 \tan^{-1} x \, dx$

17. (a) Find the area of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$

(b) Find the entire length of the cardioid  $r = a(1 + \cos \theta)$

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